Anisotropic conductivity tensor on a half-filled high Landau level

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We study two-dimensional interacting electrons in a weak perpendicular magnetic field with the filling factor $\nu \gg 1$ and in the presence of a quenched disorder. As it is known, the unidirectional charge density wave state can exist near a half-filled high Landau level at low temperatures if disorder is weak enough. We show that the existence of the unidirectional charge density wave state at temperature $T < T_c$ where T_c is the transition temperature leads to the anisotropic conductivity tensor. We find that the anisotropic part of conductivity tensor is proportional to $(T_c - T)/T_c$ below the transition in accordance with the experimental findings. The order parameter fluctuations wash out the mean-field cusp at $T = T_c$ and the conductivity tensor becomes anisotropic even above the mean-field transition temperature T_c.

I. INTRODUCTION

A two-dimensional electron gas (*2DEG*) in a perpendicular magnetic field (*H*) have remained a subject of intensive studies, both theoretical and experimental, for several decades[1]. Recently the phenomenon of sharp anisotropy of magnetoresistance near half-filled high Landau levels with Landau level index $N = 2, 3, 4$ at low temperatures have been discovered[2,3]. Right away, the magnetoresistance anisotropy have been related with the possible existence of unidirectional charge density wave (*UCDW*) state near half-filling of a high Landau level that theoretical results of Refs.[4,5,6] had been predicted in the limit $N \gg 1$. Later a bunch of theoretical and experimental papers have appeared[7]. However, only a few theoretical researches have been concerned the transport properties of the system whereas the only experimentally measured quantity is magnetoresistance.

In the limit of zero temperature the unidirectional charge density wave has strongly non-sinusoidal profile with well-defined edges[4] and the conductivity tensor can be evaluated in the framework of an effective theory for edge excitations[8]. The result of Ref.[8] is in a qualitative agreement with measured magnetoresistance behavior in the limit of zero temperature[3,9]. Later the "semicircle law" of Dykhne-Ruzin[10] have also been extended to the case of the anisotropic conductivity tensor[11].

At present a thorough analysis of temperature (*T*) dependence of magnetoresistance near the transition temperature T_c from the liquid state to the UCDW state is absent. The main objective of the paper is to present such analysis for the domain $|T_c - T|/T_c \ll 1$ where the expansion in the UCDW order parameter is legitimate. The effect of the order parameter fluctuations that are enhanced near the phase transition on the conductivity tensor both below and above the transition temperature T_c is also investigated.

We assume the presence of a *weak* random potential created by impurities near 2DEG such that the Landau level broadening $1/\tau$ is much less the spacing ω_H between Landau levels, $1/\tau \ll \omega_H$. Here the ω_H is cyclotron frequency $\omega_H = eH/m$ with e and m being the electron charge and band mass respectively (we use the units with $\hbar = 1$, $c = 1$, and $k_B = 1$).

One among the main results of the paper is the fact that the conductivity tensor σ_{ab} (we measure conductivity in units of e^2/h) of two-dimensional electrons in the UCDW state acquires anisotropic part $\sigma_{ab}^{(\text{anis})}$ proportional to temperature deviation from T_c, i.e. $\sigma_{ab}^{(\text{anis})} \propto (T_c - T)/T_c$. Other main result is that in the vicinity of the transition temperature T_c there is additional anisotropic contribution $\delta\sigma_{ab}^{(\text{anis}-\text{f})} \propto (|T_c - T|/T_c)^{-3/2}$ that we will refer as fluctuational to the conductivity tensor $\sigma_{ab}^{(0)}$ of the liquid state due to UCDW order parameter fluctuations.

We start out with an introduction to the formalism that mainly follows one introduced in the previous paper[12]. In section II the effective action of "three level model" is developed. The conductivity tensor is evaluated in section III. The effect of order parameter fluctuations is investigated in section IV. In section V the results obtained are discussed in relation with recent experiments[2,3].

Some of the results of the present paper have been published in a brief form in Ref.[13].

II. "THREE LEVEL" MODEL

A. Introduction

To start out consider the system of two-dimensional interacting electrons in the presence of a random potential $V_{\text{dis}}(\mathbf{r})$ and perpendicular magnetic field H. The parameter that characterizes the strength of the Coulomb interaction is $r_s = \sqrt{2}e^2/\varepsilon v_F$ with v_F being the Fermi velocity and ε the dielectric constant of a media. We assume that the Coulomb interaction between the electrons is weak, $r_s \ll 1$, and the magnetic field obeys the condi-

tion $Nr_s \gg 1$ where the Landau level index $N = [\nu/2]$ is the integer part of half of filling factor ν. In addition, we assume that the Landau level broadening $1/\tau$ is not too small, $1/\tau \gg \omega_H N^{-1} \ln \sqrt{2} r_s N$. In this case it is possible to construct an effective field theory for electrons on the highest partially filled Landau level by integrating out all other degrees of freedom[14,15].

Also, we consider the case when the Landau levels are spin-resolved that occurs according to Ref.[14] if $1/\tau \ll \Delta_{\mathrm{ex}} = (r_s\omega_H/\pi\sqrt{2}) \ln 2\sqrt{2}/r_s$. Therefore, the Landau level broadening should be restricted from below and from above as

$$\frac{\omega_H}{N} \ln \sqrt{2} r_s N \ll \frac{1}{\tau} \ll \frac{r_s\omega_H}{\pi\sqrt{2}} \ln \frac{2\sqrt{2}}{r_s}. \quad (1)$$

Throughout the paper we characterize the UCDW state by the order parameter $\Delta(\mathbf{Q})$. All calculations are performed under the assumption $Nr_s^2 \gg 1$. In this case the Hartree-Fock approximation is well justified[6] moreover it is known that the corrections to it are small in the parameter $a_B/l_H = 1/Nr_s^2 \ll 1$, where $a_B = \varepsilon/me^2$ is the Bohr radius and $l_H = 1/\sqrt{m\omega_H}$ the magnetic length.

B. Formalism

Thermodynamic potential of the system in hand can be written as

$$\Omega = -\frac{T}{N_r} \int \mathcal{D}[\overline{\psi}, \psi] \int \mathcal{D}[V_{\mathrm{dis}}] \, \mathcal{P}[V_{\mathrm{dis}}] \, \exp \mathcal{S}[\overline{\psi}, \psi, V_{\mathrm{dis}}]. \quad (2)$$

where action $\mathcal{S}[\overline{\psi}, \psi, V_{\mathrm{dis}}]$ is written in Matsubara representation

$$\mathcal{S} = \int d\mathbf{r} \sum_{\alpha, \omega_n, \omega_m} \overline{\psi_{\omega_n}^{\alpha,\sigma}}(\mathbf{r}) \Big[(i\omega_n + \mu - V_{\mathrm{dis}}(\mathbf{r}))\delta_{nm} - \hat{\mathcal{H}} \Big]$$
$$\times \psi_{\omega_m}^{\alpha,\sigma}(\mathbf{r}) - \frac{T}{2} \sum_{\omega_n, \omega_m, \nu_l} \int d\mathbf{r} d\mathbf{r}' \overline{\psi_{\omega_n}^{\alpha,\sigma}}(\mathbf{r}) \psi_{\omega_n-\nu_l}^{\alpha,\sigma}(\mathbf{r})$$
$$\times U_0(\mathbf{r}-\mathbf{r}') \overline{\psi_{\omega_m}^{\alpha,\sigma'}}(\mathbf{r}) \psi_{\omega_m+\nu_l}^{\alpha,\sigma'}(\mathbf{r}'). \quad (3)$$

Here, $\psi_{\omega_n}^{\alpha,\sigma}(\mathbf{r})$ and $\overline{\psi_{\omega_n}^{\alpha,\sigma}}(\mathbf{r})$ are annihilation and creation electron operators. T stands for temperature, μ chemical potential, σ and σ' spin indices, $\omega_n = \pi T(2n+1)$ fermionic frequency whereas $\nu_n = 2\pi Tn$ bosonic one. Matrix $\hat{\mathcal{H}}$ is defined as

$$\hat{\mathcal{H}} = \sum_{\alpha\nu_n} \mathcal{H}(\nu_n) I_n^\alpha, \quad (4)$$

with matrices

$$(I_n^\alpha)_{kl}^{\beta\gamma} = \delta^{\alpha\beta}\delta^{\alpha\gamma}\delta_{k-l,n} \quad (5)$$

being $U(1)$ generators. One-particle hamiltonian $\mathcal{H}$ describes a two-dimensional electron in constant perpendicular magnetic field $H = \epsilon_{ab}\partial_a A_b$ and in time-dependent

magnetic field with vector-potential $\mathbf{a}$,

$$\mathcal{H} = \frac{1}{2m}(-i\nabla - e\mathbf{A} - e\,\mathbf{a})^2. \quad (6)$$

As usual, we assume the white-noise distribution for the random potential,

$$\mathcal{P}[V_{\mathrm{dis}}(\mathbf{r})] = \frac{1}{\sqrt{2\pi g}} \exp \left(-\frac{1}{2g} \int d\mathbf{r} V_{\mathrm{dis}}^2(\mathbf{r}) \right). \quad (7)$$

In order to average over disorder we introduce N_r replicated copies of the system[16] labelled by the replica indices $\alpha = 1, \cdots, N_r$.

It is convenient to rewrite one-particle hamiltonian (6) with a help of covariant derivative

$$\mathbf{D} = \nabla - ie\mathbf{A}, \quad (8)$$

in order to extract the time-dependent vector potential $\mathbf{a}(\nu_n)$,

$$\mathcal{H} = -\frac{1}{2m}\mathbf{D}^2 + K(\nu_n), \quad (9)$$

$$K(\nu_n) = -\frac{e}{m}\mathbf{a}(\nu_n)\mathbf{D} + \frac{e^2}{2m}\sum_{\nu_m} \mathbf{a}(\nu_{n-m})\,\mathbf{a}(\nu_m). $$

C. Effective action of "three-level" model

To investigate the thermodynamic properties of electrons on the Nth Landau level one can integrate out electrons on all other Landau levels[14]. However, to find conductivity tensor projection on the single Nth Landau level is not appropriate because of covariant derivative $\mathbf{D}$ has non-zero matrix elements only for transitions between adjacent Landau levels. It is necessary therefore to consider not only the Nth Landau level alone but two adjacent ones, the $(N-1)$th and $(N+1)$th Landau levels.

Extending the projection to the Nth Landau level only of Refs.[14,15], we obtain effective action for electrons on the $(N-1)$th, Nth, and $(N+1)$th Landau level as follows

$$\mathcal{S} = \int d\mathbf{r} \sum_{\alpha, \omega_n, \omega_m} \overline{\psi_{\omega_n}^{\alpha,\sigma}}(\mathbf{r}) \Big[(i\omega_n + \mu - V_{\mathrm{dis}}(\mathbf{r}))\delta_{nm} - \hat{\mathcal{H}} \Big]$$
$$\times \psi_{\omega_m}^{\alpha,\sigma}(\mathbf{r}) - \frac{T}{2} \sum_{\omega_n, \omega_m, \nu_l} \int d\mathbf{r} d\mathbf{r}' \overline{\psi_{\omega_n}^{\alpha,\sigma}}(\mathbf{r}) \psi_{\omega_n-\nu_l}^{\alpha,\sigma}(\mathbf{r})$$
$$\times U_{\mathrm{scr}}(\mathbf{r}-\mathbf{r}') \overline{\psi_{\omega_m}^{\alpha,\sigma'}}(\mathbf{r}) \psi_{\omega_m+\nu_l}^{\alpha,\sigma'}(\mathbf{r}'). \quad (10)$$

Here $\psi_{\omega_n}^{\alpha,\sigma}(\mathbf{r})$ and $\overline{\psi_{\omega_n}^{\alpha,\sigma}}(\mathbf{r})$ are annihilation and creation operators of an electron on the $(N-1)$th, Nth, and $(N+1)$th Landau levels,

$$\psi_{\omega_n}^{\alpha,\sigma}(\mathbf{r}) = \sum_{p=N-1}^{N+1} \psi_{p\omega_n}^{\alpha,\sigma}(\mathbf{r}), \qquad \overline{\psi_{\omega_n}^{\alpha,\sigma}}(\mathbf{r}) = \sum_{p=N-1}^{N+1} \overline{\psi_{p\omega_n}^{\alpha,\sigma}}(\mathbf{r}). \quad (11)$$

The screened electron-electron interaction $U_{\rm scr}(\mathbf{r})$ has the following Fourier transform

$$U_{\rm scr}(q) = \frac{2\pi e^2}{\varepsilon q} \left[1 + \frac{2}{qa_B} \left(1 - \frac{\pi}{6\omega_H\tau} \right) \right.$$

$$\left. \times \; \left(1 - \mathcal{J}_0^2(qR_c) - 2\mathcal{J}_1^2(qR_c) \right) \right]^{-1}. \quad (12)$$

It is different from one obtained in Ref.[15,17]. The reason for that is exclusion of contributions from the $(N-1)$th and $(N+1)$th Landau level from the polarization operator.

Effective action (10) was obtained under assumptions discussed in Sec. II A. Hereafter, for reasons to be explained shortly we neglect small correction $\pi/(6\omega_H\tau) \ll 1$ in the screened electron-electron interaction (12).

D. Hartree-Fock decoupling

Effective action (10) involves electron states with spin-up and spin-down projections. Electron-electron interaction can flip electron spin. Therefore, a charge density wave state is characterized by an order parameter $\Delta_{p_1p_2}^{\sigma_1\sigma_2}(\mathbf{Q})$ that is matrix in the space of Landau level and spin indices. However, as it will be clear from discussion below, if the Landau levels are spin-resolved, i.e. $\Delta_{\rm ex} \gg \max\{T, \tau^{-1}\}$, the charge density wave state creates only on the Nth Landau level with certain spin projection. Then Landau levels with different spin projection become completely separated and can be ignored. Thus, we can consider the charge density wave order parameter to be matrix only in the space of Landau level indices. It is related with distortion of electron density on the $(N-1)$th, Nth, and $(N+1)$th Landau levels as

$$\langle \rho(\mathbf{q}) \rangle = S n_L \sum_{p_1, p_2 = N-1}^{N+1} \Delta_{p_1p_2}(\mathbf{q}) F_{p_1p_2}(\mathbf{q}), \quad (13)$$

where S stands for the area of two-dimensional electron gas and form-factor $F_{p_1p_2}(\mathbf{q})$ is defined as

$$F_{p_1p_2}(\mathbf{q}) = n_L^{-1} \sum_k \phi_{p_1k}^*(0)\phi_{p_2k}(\mathbf{q}l_H^2) \exp\left(\frac{i}{2} q_x q_y l_H^2 \right). \quad (14)$$

After Hartree-Fock decoupling of interaction term in effective action (10) (see Ref.[18]), we obtain

$$S = -\frac{N_r\Omega_\Delta}{T} + \int d\mathbf{r} \sum_{p_1,p_2} \sum_{\alpha,\omega_n,\omega_m} \overline{\psi_{p_1\omega_n}^\alpha}(\mathbf{r}) \Big[(i\omega_n + \mu$$

$$- V_{\rm dis}(\mathbf{r}))\delta_{nm} - \hat{\mathcal{H}} + \lambda_{p_1p_2}(\mathbf{r}) \Big] \psi_{p_2\omega_m}^\alpha(\mathbf{r}), \quad (15)$$

where

$$\Omega_\Delta = \frac{n_L S^2}{2} \sum_{p_i} \int \frac{d\mathbf{q}}{(2\pi)^2} U_{p_1p_2p_3p_4}(\mathbf{q})\Delta_{p_1p_4}(\mathbf{q})\Delta_{p_3p_2}(-\mathbf{q}). \quad (16)$$

Potential $\lambda_{p_1p_2}(\mathbf{r})$ in Eq. (15) appears as a consequence of distortion of uniform electron density by the charge density wave and is related with the order parameter as

$$\lambda_{p_1p_2}(\mathbf{q}) = S \sum_{p_3p_4} \frac{U_{p_3p_4p_1p_2}(\mathbf{q})}{F_{p_1p_2}(-\mathbf{q})} \Delta_{p_3p_4}(\mathbf{q}), \quad (17)$$

where $U_{p_1p_2p_3p_4}(\mathbf{q})$ denotes the generalized Hartree-Fock potential

$$U_{p_1p_2p_3p_4}(\mathbf{q}) = -n_L \Big(U_{\rm scr}(\mathbf{q})F_{p_1p_2}(\mathbf{q})F_{p_3p_4}(-\mathbf{q}) \quad (18)$$

$$- \int \frac{d\mathbf{p}}{(2\pi)^2 n_L} e^{-i\mathbf{q}\mathbf{p}l_H^2} U_{\rm scr}(\mathbf{p})F_{p_1p_4}(\mathbf{p})F_{p_3p_2}(-\mathbf{p}) \Big).$$

E. Average over disorder

After standard average over the random potential $V_{\rm dis}(\mathbf{r})$ (see Ref.[19]), effective action (15) becomes

$$S = -\frac{N_r\Omega_\Delta}{T} + \int d\mathbf{r}\psi^\dagger(\mathbf{r}) \Big(i\omega + \mu - \hat{\mathcal{H}} + \check{\lambda} + iQ \Big) \psi(\mathbf{r})$$

$$- \frac{1}{2g} \int d\mathbf{r}\, {\rm tr}\, Q^2(\mathbf{r}), \quad (19)$$

where we introduce new field $Q(\mathbf{r})$, that is unitary matrix in Matsubara and replica spaces. For convenience we use the following notation

$$\psi^\dagger\check{\lambda}\psi = \sum_{p_1p_2} \sum_{\alpha,\omega_n} \overline{\psi_{p_1\omega_n}^\alpha}(\mathbf{r})\lambda_{p_1p_2}(\mathbf{r})\psi_{p_2\omega_n}^\alpha(\mathbf{r}). \quad (20)$$

Let us recall that action (19) at zero temperature, i.e. for $\omega_n \to 0$, and in the absence of the induced potential $\check{\lambda}(\mathbf{r})$ and the time-dependent vector-potential $\mathbf{a}$ has the following saddle-point solution

$$Q_{\rm sp} = V^{-1}P_{\rm sp}V, \qquad (P_{\rm sp})_{nm}^{\alpha\beta} = P_{\rm sp}^n\delta_{nm}\delta^{\alpha\beta}, \quad (21)$$

where V is arbitrary global unitary rotation and $P_{\rm sp}^n$ obeys the equation

$$P_{sp}^n = igG^{\omega_n}(\mathbf{r},\mathbf{r}), \qquad G^{\omega_n}(\mathbf{r},\mathbf{r}') = \sum_{p=N-1}^{N+1} G_p^{\omega_n}(\mathbf{r},\mathbf{r}').$$

$$(22)$$

Here Green function $G_p^{\omega_n}(\mathbf{r},\mathbf{r}')$ is as follows

$$G_p^n(\mathbf{r},\mathbf{r}') = \sum_k \phi_{pk}^*(\mathbf{r})G_p(\omega_n)\phi_{pk}(\mathbf{r}'), \quad (23)$$

$$G_p(\omega_n) = [i\omega_n + \mu_N + \epsilon_N - \epsilon_p + iP_{\rm sp}^n]^{-1},$$

where chemical potential μ_N is measured from the Nth Landau level. The $\epsilon_p = \omega_H(p+1/2)$ and $\phi_{pk}(\mathbf{r})$ are the eigenvalues and eigenfunctions of the hamiltonian $\mathcal{H}$, and k denotes pseudomomentum. In the case of weak disorder, $\omega_H\tau \gg 1$, solution of Eq.(22) yields

$$P_{\rm sp}^n = \frac{{\rm sign}\,\omega_n}{2\tau}, \qquad \frac{1}{2\tau} = \sqrt{gn_L}, \quad (24)$$

with $n_L = 1/2\pi l_H^2$.

The fluctuations of the V field are responsible for the localization corrections to the conductivity (in the weak localization regime they correspond to the maximally crossed diagrams). However, in the considered case, these corrections are of the order of $\ln N/N \ll 1$ and, therefore, can be neglected. For this reason we simply put $V = 1$.

The presence of the induced potential $\check{\lambda}(\mathbf{r})$ and the time-dependent vector potential $\mathbf{a}$ results in a shift of the saddle-point value (24) due to the coupling to the fluctuations $\delta P = P - P_{\rm sp}$ of the P field. The corresponding effective action for the δP field follows from Eq. (16) after integrating out fermions:

$$\mathcal{S} = \int d\mathbf{r}\, \mathrm{tr}\ln G^{-1} - \frac{N_r \Omega_\Delta}{T} - \frac{1}{2g}\int d\mathbf{r}\, \mathrm{tr}(P_{\rm sp} + \delta P)^2$$
$$+ \int d\mathbf{r}\, \mathrm{tr}\ln\left[1 + (i\delta P + \hat{K} + \check{\lambda})G\right]. \quad (25)$$

Finally, the thermodynamic potential can be written as

$$\Omega = -\frac{T}{N_r}\ln\int \mathcal{D}[\delta P] I[\delta P]\exp\mathcal{S}, \quad (26)$$

where following Ref.[20] the integration measure $I[\delta P]$ is given by

$$\ln I[\delta P] = -\frac{1}{(\pi\rho)^2}\int\sum_{nm}^{\alpha\beta}[1 - \Theta(nm)]\,\delta P_{nn}^{\alpha\alpha}\delta P_{mm}^{\beta\beta}, \quad (27)$$

with ρ being the thermodynamic density of states and $\Theta(x)$ the Heaviside step function.

The quadratic in δP part of the action (25) together with the contribution (27) from the integration measure determine the propagator of the δP fields (see Ref.[15] for details)

$$\langle \delta P_{m_1 m_2}^{\alpha\beta}(\mathbf{q})\delta P_{m_3 m_4}^{\gamma\delta}(-\mathbf{q})\rangle \quad (28)$$
$$= \frac{g\delta_{m_1 m_4}\delta_{m_2 m_3}\delta^{\alpha\delta}\delta^{\beta\gamma}\Theta(m_1 m_3)}{1 + g\pi^{\omega_{m_1}}(\omega_{m_3} - \omega_{m_1}, \mathbf{q})} - \frac{2\left[1 - \Theta(m_1 m_3)\right]}{(\pi\rho)^2}$$
$$\times \frac{g\delta_{m_1 m_2}\delta^{\alpha\delta}}{1 + g\pi^{\omega_{m_1}}(0, \mathbf{q})}\frac{g\delta_{m_3 m_4}\delta^{\delta\gamma}}{1 + g\pi^{\omega_{m_3}}(0, \mathbf{q})},$$

where the bare polarization operator $\pi^{\omega_m}(\nu_n, \mathbf{q})$ involves Green functions for the $(N-1)$th, Nth, and $(N+1)$th Landau levels only

$$\pi^{\omega_m}(\nu_n, \mathbf{q}) = \sum_{p_1 p_2}\pi_{p_1 p_2}^{\omega_m}(\nu_n, \mathbf{q}) = -n_L\sum_{p_1 p_2}G_{p_2}(\omega_m)$$
$$\times\ G_{p_1}(\omega_m + \nu_n)F_{p_1 p_2}(\mathbf{q})F_{p_2 p_1}(-\mathbf{q}). \quad (29)$$

F. Thermodynamic potential. Second order contribution

In the absence of the time-dependent vector potential $\mathbf{a}$ effective action (25) should contain only $\Delta_{NN}(\mathbf{q}) \equiv \Delta(\mathbf{q})$ order parameter in the limit $\max\{T, \tau^{-1}\} \ll \Delta_{\rm ex} \ll \omega_H$.

To demonstrate it, we find the second order contribution to the thermodynamic potential for $\mathbf{a} = 0$.

Performing evaluation similar to one presented in Ref.[12], we obtain

$$\Omega = \Omega^{(0)} + \Omega^{(2)} + \cdots, \quad (30)$$

where

$$\Omega^{(0)}(\mu) = \int d\mathbf{r}\, \mathrm{tr}\ln G^{-1} - \frac{1}{2g}\int d\mathbf{r}\, \mathrm{tr}\,P_{\rm sp}^2 \quad (31)$$

is the thermodynamic potential of the liquid state and

$$\Omega^{(2)} = \frac{n_L S^2}{2T}\sum_{p_1\cdots p_4}\int\frac{d\mathbf{q}}{(2\pi)^2}\left[U_{p_1 p_2 p_3 p_4}(\mathbf{q})\right. \quad (32)$$
$$- T\sum_{\omega_n}\sum_{p_5\cdots p_8}\frac{U_{p_1 p_2 p_5 p_6}(\mathbf{q})}{F_{p_5 p_6}(\mathbf{q})}\frac{U_{p_3 p_4 p_7 p_8}(-\mathbf{q})}{F_{p_7 p_8}(-\mathbf{q})}$$
$$\times\left(\delta_{p_5 p_8}\delta_{p_6 p_7} - \frac{g\pi_{p_8 p_7}^{\omega_n}(0, \mathbf{q})}{1 + g\pi^{\omega_n}(0, \mathbf{q})}\right)\pi_{p_5 p_6}^{\omega_n}(0, \mathbf{q})\bigg]$$
$$\times\ \Delta_{p_1 p_2}(\mathbf{q})\Delta_{p_3 p_4}(-\mathbf{q})$$

is the contribution to the thermodynamic potential quadratic in the order parameter $\Delta_{p_1 p_2}(\mathbf{q})$.

It is worthwhile to mention that polarization operators $\pi_{p_1 p_2}^{\omega_n}(\nu_n, \mathbf{q})$ obey the following hierarchy with respect to small parameter, $\max\{T, \tau^{-1}\}/\omega_H \ll 1$,

$$\pi_{p_1 p_2}^{\omega_n} \sim \begin{cases} \mathcal{O}(1), & p_1 = p_2 = N, \\ \mathcal{O}\left(\frac{\max\{T, \tau^{-1}\}}{\omega_H}\right), & p_1 = N\,\text{or}\,p_2 = N, \\ \mathcal{O}\left(\left[\frac{\max\{T, \tau^{-1}\}}{\omega_H}\right]^2\right), & p_1 \neq N\,\text{and}\,p_2 \neq N. \end{cases} \quad (33)$$

According to the hierarchy (33) we can write

$$\pi_{p_1 p_2}^{\omega_n}(0, \mathbf{q}) \approx \pi_0^{\omega_n}(0, q)\delta_{p_1 N}\delta_{p_2 N}, \quad (34)$$

where we introduce $\pi_0^{\omega_n}(0, q) \equiv \pi_{NN}^{\omega_n}(0, q)$. Thus from Eq.(32) we obtain

$$\Omega^{(2)} = \frac{n_L S^2}{2T}\sum_{p_1\cdots p_4}\int\frac{d\mathbf{q}}{(2\pi)^2}\left[U_{p_1 p_2 p_3 p_4}(\mathbf{q})\right. \quad (35)$$
$$+ T\sum_{\omega_n}\frac{n_L G_N^2(\omega_n)}{1 + g\pi_0^{\omega_n}(0, \mathbf{q})}U_{p_1 p_2 NN}(\mathbf{q})U_{p_3 p_4 NN}(-\mathbf{q})\bigg]$$
$$\times\ \Delta_{p_1 p_2}(\mathbf{q})\Delta_{p_3 p_4}(-\mathbf{q}).$$

To find the possible non-zero order parameters $\Delta_{p_1 p_2}(\mathbf{q})$, we should diagonalize the 9×9 matrix in Eq.(35). Fortunately, non-trivial part of Eq.(35) can be written as

$$\delta\Omega^{(2)} = \frac{n_L S^2}{2T}\int\frac{d\mathbf{q}}{(2\pi)^2}T_0(q)\left(\Delta(\mathbf{q}), \frac{T_1(q)}{T_0(q)}\varphi(\mathbf{q})\right)$$
$$\times\begin{pmatrix} a(q) & a(q) \\ a(q) & a(q) + 2\xi(q) \end{pmatrix}\begin{pmatrix} \Delta(-\mathbf{q}) \\ \frac{T_1(q)}{T_0(q)}\varphi(-\mathbf{q}) \end{pmatrix}. \quad (36)$$

Here $\varphi(\mathbf{q})$ involves a linear combination of all order parameters $\Delta_{p_1 p_2}(\mathbf{q})$ except $\Delta(\mathbf{q})$. Characteristic energies $T_0(q)$ and $T_1(q)$ is related with Hartree-Fock potential (18) as follows

$$T_0(q) = \frac{U_{NNNN}(q)}{4}, \qquad T_1(q) = e^{i\phi}\frac{U_{N,N\pm1,NN}(\mathbf{q})}{4}, \tag{37}$$

where ϕ denotes angle of vector $\mathbf{q}$ with respect to the x axis. We emphasize that quantity $T_1(q)$ depends only on the absolute value q of vector $\mathbf{q}$. Matrix element $a(q)$ is given by

$$a(T, \tau^{-1}, q) = 1 + 4T \sum_{\omega_n} \frac{n_L T_0(q) G_N^2(\omega_n)}{1 + g\pi^{\omega_n}(0, q)}, \tag{38}$$

whereas function $\xi(q)$ is defined as

$$\xi(q) = \frac{T_0(q)}{2T_1(q)} - \frac{1}{2}. \tag{39}$$

The eigenvalues of the 2×2 matrix in Eq.(36) can be easily found

$$\lambda_\pm(q) = a(q) + \hat{\xi}(q) \pm \sqrt{[a(q)]^2 + [\xi(q)]^2} \tag{40}$$

As one can check, the eigenvalue $\lambda_+(q)$ has the same sign as $\xi(q)$ for all values of $a(q)$ whereas the eigenvalue $\lambda_-(q)$ changes its sign at point $a(q) = 0$. Therefore, the instability appears at the same condition as if we consider only one charge density wave order parameter $\Delta(\mathbf{q})$ as it has usually done [4,5,6,12]. According to the result derived in Appendix A, characteristic energy $T_1(q)$ is of the order of $T_0(q)/N \ll T_0(q)$. By using the condition $\xi(q) \gg a(q)$, we find therefore

$$\begin{aligned}\Omega^{(2)} &= \frac{n_L S^2}{2T} \int \frac{d\mathbf{q}}{(2\pi)^2} T_0(q) \left[a(q) \left(1 - \frac{T_1(q)}{T_0(q)} a(q) \right) \right. \\ &\quad \times \left. \Delta_-(\mathbf{q})\Delta_-(-\mathbf{q}) + \frac{T_1(q)}{T_0(q)}\Delta_+(\mathbf{q})\Delta_+(-\mathbf{q}) \right], \end{aligned} \tag{41}$$

where

$$\Delta_-(\mathbf{q}) = \Delta(\mathbf{q}) - \left(\frac{T_1(q)}{T_0(q)}\right)^2 a(q)\varphi(\mathbf{q}), \tag{42}$$
$$\Delta_+(\mathbf{q}) = \varphi(\mathbf{q}) + a(q)\Delta(\mathbf{q}).$$

Minimum of the free energy[21] is reached at $\Delta_+(q) = 0$. Neglecting the difference of the order of $\mathcal{O}(N^{-2})$ between $\Delta_-(q)$ and $\Delta(q)$, we obtain finally

$$\begin{aligned}\Omega^{(2)} &= \frac{n_L S^2}{2T} \int \frac{d\mathbf{q}}{(2\pi)^2} T_0(q) a(q) \left(1 - \frac{T_1(q)}{T_0(q)} a(q) \right) \\ &\quad \times \Delta(\mathbf{q})\Delta(-\mathbf{q}). \end{aligned} \tag{43}$$

Thus, the fact that the order parameters $\Delta_{p_1 p_2}(\mathbf{q})$ with p_1 and p_2 different from N can exist leads to correction of the order of $\mathcal{O}(N^{-1})$. Later on we assume therefore that

$$\Delta_{p_1 p_2}(\mathbf{q}) = \Delta(\mathbf{q})\delta_{p_1 N}\delta_{p_2 N}. \tag{44}$$

G. "Three-level" model

The results of the previous section allows us to establish finally an effective action for the "three-level" model.

According to definition (17), charge density wave on the Nth Landau level with the order parameter $\Delta(\mathbf{q})$ results in the induced potential $\lambda_{N,N\pm1}(\mathbf{q})$, scattering electrons from the Nth Landau level to the $(N\pm1)$th Landau level. However, the induced potential is of the order of $T_1(q)$ and, consequently, leads to small corrections of the order of $\mathcal{O}(N^{-1})$. For the reasons to be explained shortly, we write

$$\lambda_{p_1 p_2}(\mathbf{q}) = SU(q)F_N^{-1}(q)\Delta(\mathbf{q})\delta_{p_1 N}\delta_{p_2 N}. \tag{45}$$

Finally, the effective action for the "three-level" model becomes

$$\begin{aligned}\mathcal{S}_{TL}[\delta P] &= \int d\mathbf{r}\, \mathrm{tr}\ln G^{-1} - \frac{N_r \Omega_\Delta}{T} \\ &\quad - \frac{1}{2g}\int d\mathbf{r}\, \mathrm{tr}(P_{\mathrm{sp}} + \delta P)^2 \\ &\quad + \int d\mathbf{r}\, \mathrm{tr}\ln\Big[1 + (i\delta P + \hat{K} + P_N \lambda P_N)G\Big], \end{aligned} \tag{46}$$

where

$$\begin{aligned}P_N(\mathbf{r}_1, \mathbf{r}_2) &= \sum_k \phi_{Nk}^*(\mathbf{r}_2)\phi_{Nk}(\mathbf{r}_1) \\ &= n_L \exp\left(i\frac{(y_1 - y_2)(x_1 + x_2)}{2l_H^2}\right) \\ &\quad \times \exp\left(-\frac{|\mathbf{r}_1 - \mathbf{r}_2|^2}{4l_H^2}\right) L_N\left(\frac{|\mathbf{r}_1 - \mathbf{r}_2|^2}{2l_H^2}\right) \end{aligned} \tag{47}$$

is the projection operator on the Nth Landau level ($L_N(x)$ denotes the Laguerre polynomial) and

$$\Omega_\Delta = \frac{n_L S^2}{2}\int \frac{d\mathbf{q}}{(2\pi)^2} U(q)\Delta(\mathbf{q})\Delta(-\mathbf{q}), \tag{48}$$

Here, for a brevity we introduce $U(q) \equiv U_{NNNN}(q)$.

III. CONDUCTIVITY OF THE UCDW STATE AT $T_c - T \ll T_c$

A. Conductivity tensor σ_{ab}

Effective action (46) allows us to evaluate conductivity of the system in the CDW state. As the most interesting case we consider the half-filled Nth Landau level where at $T < T_c$ the UCDW state exists. The order parameter is[18]

$$\Delta(\mathbf{q}) = \frac{(2\pi)^2}{S}\Delta\Big[\delta(\mathbf{q} - \mathbf{Q}_0) + \delta(\mathbf{q} - \mathbf{Q}_0)\Big], \tag{49}$$

where vector $\mathbf{Q}_0$ that determines period and direction of the UCDW state can be oriented along spontaneously
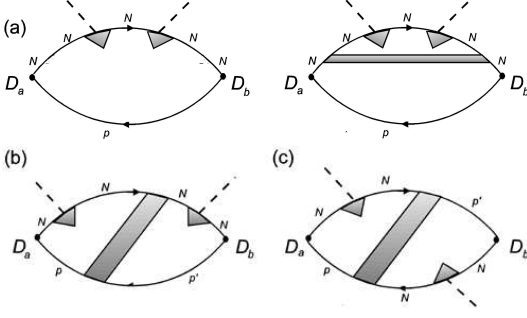
FIG. 1: Diagrams for corrections to the conductivity tensor σ_{ab}. Solid line denote Green function, $N/p/p'$ Landau level indices, dashed line the induced potential $\lambda(\boldsymbol{r})$, and shaded block the impurity ladder.

FIG. 2: Impurity ladder. Frequency $\omega_n + \nu_n$ runs from right to left, whereas ω_n runs from left to right.

chosen direction. Usually, its direction is fixed either by intrinsic anisotropy of the system or by small magnetic field applied parallel to 2DEG[22,23,24,25,26]. We assume that the vector $\mathbf{Q}_0$ is directed at an angle ϕ with respect to the x axis. Let us recall that the absolute value of the vector $\mathbf{Q}_0$ equals $Q_0 = r_0/R_c$ with $r_0 \approx 2.4$ being the first zero of the Bessel function $\mathcal{J}_0(x)$.

The conductivity tensor $\sigma_{ab}(\nu_n, q)$ at $q = 0$ can be found after integration over $\delta P(\mathbf{r})$ fields as the second derivative of logarithm of the effective action with respect to spatially constant time-dependent vector potential $\mathbf{a}(\nu_n)$,

$$\sigma_{ab}(\nu_n) = \frac{\pi T}{SN_r\nu_n}\frac{\delta^2}{\delta a_a(\nu_n)\delta a_b(-\nu_n)} \quad (50)$$
$$\times \ln \int \mathcal{D}\delta P\, I[\delta P]\exp\mathcal{S}_{TL}[\delta P]\Big|_{\mathbf{a}=0}.$$

It is worthwhile to mention that Eq.(50) corresponds to the term $\mathbf{j}(\nu_n)\mathbf{a}(-\nu_n)$ with $\mathbf{j}(\nu_n)$ being current density in the effective action for the vector potential $\mathbf{a}(\nu_n)$. As one can check by inspection, contribution to the conductivity tensor of the first order in the order parameter $\Delta(\mathbf{q})$ vanishes. It occurs because the UCDW state appears at non-zero vector $\mathbf{Q}_0$. Thus, the first non-vanishing contribution to the conductivity tensor of the UCDW state is of the second order in the order parameter Δ.

In order to find it, we expand the effective action $\mathcal{S}_{TL}[\delta P]$ to the second order in both the induced potential $\lambda(\mathbf{r})$ and the $\hat{K}$. Then, we integrate over $\delta P(\mathbf{r})$ fields. We do not present the explicit calculations here since they are similar to ones presented in the Ref.[12]. We mention that there are three contributions of different structure to the conductivity tensor of the UCDW state. Diagrams for them are shown in Fig. 1.

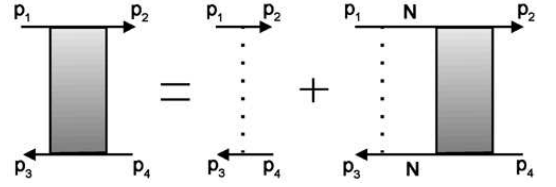The first and the second diagrams (Fig. 1(a)) corre-

spond to the following contribution

$$\sigma_{ab}^{(a)}(\nu_n) = -\frac{8\pi\omega_H}{m\nu_n}T_0^2\Delta^2 T\sum_{\omega_n}\frac{G_N^3(\omega_n)}{1 + g\pi_0^{\omega_n}(0,0)}$$
$$\times \sum_p \frac{D_{Np}^a D_{pN}^b G_p(\omega_n + \nu_n)}{(1 + g\pi_0^{\omega_n}(0,Q_0))^2}. \quad (51)$$

Here D_{Np}^a denotes matrix element of the covariant derivative

$$D_{Np}^a = \int d\mathbf{r}\phi_{Nk}^*(\mathbf{r})D_a\phi_{pk}(\mathbf{r}) = \sqrt{n_L}\Big[\delta_{p,N-1}\beta^a\sqrt{N}$$
$$+ \delta_{p,N+1}\gamma^a\sqrt{N+1}\Big], \quad (52)$$

where

$$\gamma^x = i, \quad \gamma^y = 1, \quad \beta^x = -i, \quad \beta^y = 1. \quad (53)$$

We note that Eq.(51) contains only isotropic contribution to the conductivity tensor, i.e. $\sigma_{xx}^{(a)} = \sigma_{yy}^{(a)}$ and $\sigma_{xy}^{(a)} = -\sigma_{yx}^{(a)}$ for some direction of the UCDW.

The third diagram (Fig. 1(b)) is given by

$$\sigma_{ab}^{(b)}(\nu_n) = \frac{8\pi\omega_H}{\nu_n m}T_0^2\Delta^2 T\sum_{\omega_n}\sum_{pp'}\frac{G_N^4(\omega_n + \nu_n)}{(1 + g\pi_0^{\omega_n}(0,Q_0))^2}$$
$$\times D_{pN}^a D_{Np'}^b G_p(\omega_n)G_{p'}(\omega_n)\Big[I_{Npp'N}(\mathbf{Q}_0)$$
$$+ \frac{n_L G_N(\omega_n)G_N(\omega_n + \nu_n)}{1 + g\pi_0^{\omega_n}(\nu_n, Q_0)}$$
$$\times I_{NpNN}(\mathbf{Q}_0)I_{NNp'N}(\mathbf{Q}_0)\Big]. \quad (54)$$

Symbol $I_{p_1p_2p_3p_4}(\mathbf{Q})$ denotes the impurity ladder in the Landau level index representation (see Fig. 2)

$$I_{p_1p_2p_3p_4}(\mathbf{Q}) = g\int_q F_{p_1p_2}(\mathbf{q})F_{p_3p_4}(-\mathbf{q})\exp\left(-i\mathbf{q}\mathbf{Q}l_H^2\right). \quad (55)$$

Evaluation of $I_{p_1p_2p_3p_4}(\mathbf{Q}_0)$ is given in Appendix B. For convenience we present the results for quantities $I_{p_1p_2p_3p_4}(\mathbf{Q}_0)$ in the Table I. As one can see, the $I_{NNNpN}(\mathbf{Q}_0)$, $I_{NpNNN}(\mathbf{Q}_0)$ and $I_{Npp'N}(\mathbf{Q}_0)$ are proportional to $\mathcal{J}_0(r_0) = 0$. Thus, the contribution (54) vanishes

$$\sigma_{ab}^{(b)}(\nu_n) = 0. \quad (56)$$

$I_{N,N-1,N-1,N}$	$=$	$gn_L\mathcal{J}_0^2(r_0)$		$I_{N,N+1,N+1,N}$	$=$	$gn_L\mathcal{J}_0^2(r_0)$
$I_{N,N-1,N,N+1}$	$=$	$gn_L\mathcal{J}_1^2(r_0)$		$I_{N,N+1,N,N-1}$	$=$	$gn_L\mathcal{J}_1^2(r_0)$
$I_{N,N,N-1,N}$	$=$	$gn_Le^{i\phi}\mathcal{J}_0(r_0)\mathcal{J}_1(r_0)$		$I_{N,N+1,N,N}$	$=$	$gn_Le^{-i\phi}\mathcal{J}_0(r_0)\mathcal{J}_1(r_0)$
$I_{N,N-1,N,N}$	$=$	$-gn_Le^{i\phi}\mathcal{J}_0(r_0)\mathcal{J}_1(r_0)$		$I_{N,N,N+1,N}$	$=$	$-gn_Le^{-i\phi}\mathcal{J}_0(r_0)\mathcal{J}_1(r_0)$
$I_{N,N-1,N,N-1}$	$=$	$gn_Le^{2i\phi}\mathcal{J}_1^2(r_0)$		$I_{N,N+1,N,N+1}$	$=$	$gn_Le^{-2i\phi}\mathcal{J}_1^2(r_0)$
$I_{N,N-1,N+1,N}$	$=$	$gn_Le^{2i\phi}\mathcal{J}_1(r_0)\mathcal{J}_0(r_0)$		$I_{N,N+1,N-1,N}$	$=$	$gn_Le^{-2i\phi}\mathcal{J}_1(r_0)\mathcal{J}_0(r_0)$

TABLE I: Expressions for quantities $I_{p_1p_2p_3p_4}(\mathbf{Q}_0)$ involved in Eqs.(51),(54) and (57).

The last diagram (Fig. 1(c)) can be written as

$$
\sigma_{ab}^{(c)}(\nu_n) = \frac{8\pi\omega_H}{\nu_n m}T_0^2\Delta^2 T\sum_{\omega_n}\sum_{pp'}\frac{G_N^2(\omega_n)G_N^2(\omega_n+\nu_n)}{(1+g\pi_0^{\omega_n}(0,Q_0))}
$$
$$
\times\ \frac{G_p(\omega_n)G_{p'}(\omega_n+\nu_n)}{(1+g\pi_0^{\omega_n+\nu_n}(0,Q_0))}
$$
$$
\times\ \frac{D_{pN}^a D_{p'N}^b I_{NpNp'}(\mathbf{Q}_0)}{1+g\pi_0^{\omega_n}(\nu_n,Q_0)}. \tag{57}
$$

We note that terms in the sum over Landau level indices with $p=p'=N\pm 1$ lead to the anisotropic contribution. Terms with $p=N\pm 1$ and $p'=N\mp 1$ result in the isotropic contribution.

B. Anisotropic contribution $\sigma_{ab}^{(anis)}$

We start our analysis of general expressions obtained in the previous section from Eq.(57) that contains the anisotropic contribution to the conductivity tensor. Taking into account only terms with $p=p'=N\pm 1$ in the sum over Landau level indices involved in Eq.(57) we obtain

$$
\left.\begin{array}{c}\sigma_{xx}^{(anis)}\\[2pt]\sigma_{yy}^{(anis)}\end{array}\right\} = \mp 4\pi N\mathcal{J}_1^2(r_0)h\left(\frac{1}{4\pi T_c\tau}\right)\Delta^2\cos[2\phi], \tag{58}
$$

and

$$
\left.\begin{array}{c}\sigma_{xy}^{(anis)}\\[2pt]\sigma_{yx}^{(anis)}\end{array}\right\} = 4\pi N\mathcal{J}_1^2(r_0)h\left(\frac{1}{4\pi T_c\tau}\right)\Delta^2\sin[2\phi]. \tag{59}
$$

Function $h(z)$ is given as

$$
h(z) = 5z^3\frac{\zeta\left(6,\frac{1}{2}+z\right)}{\left[\zeta\left(2,\frac{1}{2}+z\right)\right]^2} = \begin{cases}\dfrac{4\pi^2}{3}z^3, & z\ll 1,\\[8pt] 1-\dfrac{3}{z}, & z\gg 1,\end{cases} \tag{60}
$$

where $\zeta(k,z)$ denotes the generalized Riemann zeta-function[27]. Function $h(z)$ increase monotonically from 0 to 1, as it is shown in Fig. 3.

Eqs.(58) and (59) constitute one of the main results of the present paper. Anisotropic contributions (58) and (59) is seemed to be proportional to $(T_c-T)/T_c$ since in the Landau theory the order parameter $\Delta\propto\sqrt{(T_c-T)/T_c}$ (see Ref.[12] for explicit expression).

The $\sigma_{xx}^{(anis)}$ as the function of angle ϕ has the minimum at $\phi=0$ that corresponds to the vector $\mathbf{Q}_0$ directed along the x axis. We note that the modulation of electron density along the y axis is absent in this case. At $\phi=0$ conductivity $\sigma_{yy}^{(anis)}$ is positive whereas $\sigma_{xx}^{(anis)}$ is negative, moreover, they have the same absolute values. It leads to the statement that the conductivity σ_{xx} (along the electron density modulation) is less than conductivity σ_{yy} (across the modulation). We emphasize that at $\phi=0$ conductivity $\sigma_{xy}^{(anis)}$ vanishes.

If the vector $\mathbf{Q}_0$ is oriented at angle $\phi=\pi/4$ with respect to the x axis conductivities $\sigma_{xx}^{(anis)}$ and $\sigma_{yy}^{(anis)}$ vanish due to the symmetry between the x and y axes. Vice versa, conductivity $\sigma_{xy}^{(anis)}$ reaches the maximum. It is worthwhile to mention that anisotropic contributions to the conductivity of UCDW state are proportional to N as the conductivity of the liquid state[1].

C. Isotropic contribution $\sigma_{ab}^{(isot)}$

Eqs.(51) and (57) allows us to find also the isotropic contribution to the conductivity tensor at $T_c-T\ll T_c$. Taking into account Eq.(51) and terms with $p=N\pm 1$ and $p'=N\mp 1$ in the sum over Landau level indices in Eq.(57), we obtain the following isotropic contribution to conductivity σ_{xx}

$$
\delta\sigma_{xx}^{(isot)} = -4\pi N h_{xx}\left(\frac{1}{4\pi T_c\tau}\right)\Delta^2, \tag{61}
$$

where function $h_{xx}(z)$ is given by

$$
h_{xx}(z) = \mathcal{J}_1^2(r_0)h(z) + \frac{1}{2z\zeta\left(2,\frac{1}{2}+z\right)} \tag{62}
$$
$$
\times\ \left[1-\frac{\operatorname{Re}\psi'\left(\frac{1}{2}+(1-i)z\right)}{\zeta\left(2,\frac{1}{2}+z\right)}\right].
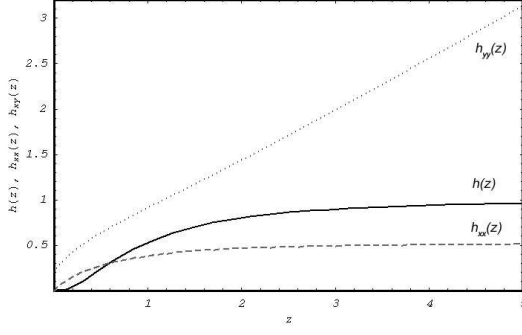$$

FIG. 3: Functions $h(z)$, $h_{xx}(z)$ and $h_{xy}(z)$.

Here $\psi(z)$ stands for the Euler di-gamma function. The $h_{xx}(z)$ has asymptotic expressions as

$$
h_{xx}(z) = \begin{cases}
\dfrac{2}{3}z, & z \ll 1, \\
\mathcal{J}_1^2(r_0) + \dfrac{1}{4} - \dfrac{3\mathcal{J}_1^2(r_0) - \frac{1}{4}}{z}, & z \gg 1.
\end{cases} \tag{63}
$$

As one can see from Eq.(61), isotropic contribution $\delta\sigma_{xx}^{(\text{isot})}$ to conductivity σ_{xx} is of the same order as the anisotropic contribution.

The isotropic contribution to conductivity σ_{xy} is as follows

$$
\delta\sigma_{xy}^{(\text{isot})} = -8\pi^2 N \frac{T_c}{\omega_H} h_{xy}\left(\frac{1}{4\pi T_c \tau}\right)\Delta^2, \tag{64}
$$

where

$$
\begin{aligned}
h_{xy}(z) =& \left[\mathcal{J}_1^2(r_0)\left[4z^4\zeta\left(5, \frac{1}{2} + z\right) + z^3\zeta\left(4, \frac{1}{2} + z\right)\right]\right. \\
& - 2h_{xx}(z) + z\,\mathrm{Im}\,\psi'\left(\frac{1}{2} + (1-i)z\right) \\
& \left. - \mathrm{Im}\,\psi\left(\frac{1}{2} + (1-i)z\right)\right] \\
& \times \frac{1}{z\left[\zeta\left(2, \frac{1}{2} + z\right)\right]^2}.
\end{aligned} \tag{65}
$$

The function $h_{xy}(z)$ has the following asymptotic expressions in the limits of small and large z

$$
h_{xy}(z) = \begin{cases}
\dfrac{2}{\pi^2}\left(1 - \dfrac{\pi^2 - 2\psi''(\frac{1}{2})}{\pi^2}z\right), & z \ll 1, \\
\left(\dfrac{\pi}{4} - \dfrac{2\mathcal{J}_1^2(r_0)}{3}\right)z, & z \gg 1.
\end{cases} \tag{66}
$$

We mention that the isotropic contribution $\sigma_{xy}^{(\text{isot})}$ contains additional small factor $\max\{T_c, \tau^{-1}\}/\omega_H$ compared to the others. Results (61) and (64) are one of the main results of the paper.

IV. EFFECT OF THE ORDER PARAMETER FLUCTUATIONS ON THE CONDUCTIVITY TENSOR

A. Order parameter fluctuations

The order parameter $\Delta(\mathbf{r})$ has meaning of the saddle-point solution for a plasmon field that appears in the Hubbard-Stratonovich transformation[28] of the screened electron-electron interaction. The expansion of such physical quantities as free energy and linear response in the order parameter series can be justified if fluctuations of the order parameter can be neglected. As it was shown[12], fluctuations of the order parameter results in the first order transition from the liquid state to the UCDW state at temperature $T_c - \delta T_c$ where $\delta T_c/T_c \propto N^{-2/3} \ll 1$. In the present section we investigate the effect of the fluctuations on the conductivity tensor above and below the mean-field transition.

In the previous section we assumed that the direction of the CDW vector $\mathbf{Q}_0$ is fixed by intrinsic anisotropy of crystal or by applied parallel to 2DEG small magnetic field. However, the functional dependence of anisotropy term in the hamiltonian was insignificant for mean-field results obtained above. Now it should be concretized. Experimental research of the anisotropy that determines the direction along which the UCDW creates has been performed in a number of papers[22,23,24,25,26]. The results obtained can be explained if one suggests that the Hartree-Fock potential $U(\mathbf{Q})$ involves terms proportional to $\cos 2\phi$ and $\cos 4\phi$. We mention that the term $\cos 2\phi$ can be derived when small magnetic field parallel to 2DEG applied[29]. However, without parallel magnetic field the term $\cos 2\phi$ is restricted by the symmetry of bulk $GaAs$ crystal. To date its physical origin is unknown[24]. As experimentally proven[26], coefficient of the $\cos 2\phi$ term depends on the density n of electrons. Moreover, at some certain value n_* of the electron density it vanishes and next term proportional to $\cos 4\phi$ becomes important. Below we restrict our discussion to the general case $n \neq n_*$. We note that the typical value of the anisotropy energy E_A is of the order of $1\,mK$ per electron as it is obtained from experiment[24]. In order to take into account the anisotropy quantitatively we perform the following substitution (see Eq.(37))

$$
T_0(Q) \to T_0(Q) + E_A \frac{1 - \cos 2\phi}{2} \tag{67}
$$

near $Q = Q_0$. We note that the expression above has minimum at $\phi = 0$.

At $T > T_c$ the UCDW order parameter is zero in average $\langle\Delta\rangle = 0$ but the average of its square is non-zero $\langle\Delta^2\rangle \neq 0$. It results in additional contribution to the conductivity tensor of the liquid state. It is worthwhile to mention that the contribution discussed above is analogous to one for normal metal due to superconducting paring above critical temperature[30].

The additional contribution to the conductivity tensor due to the order parameter fluctuations can be found with a help of the substitution $\langle \Delta(\mathbf{Q})\Delta(-\mathbf{Q})\rangle$ for Δ^2 in Eqs.(58), (59), (61) and (64) and averaging over all possible vectors $\mathbf{Q}$. The Green function of the order parameter is as follows (see Ref.[12])

$$\langle \Delta(\mathbf{Q})\Delta(-\mathbf{Q})\rangle = \frac{T_c}{4T_0(Q_0)n_L}\left[\frac{T-T_c}{T_c} + \gamma\left(\frac{1}{4\pi T_c\tau}\right)\right.$$
$$\times \left.(Q-Q_0)^2 R_c^2 + \eta\sin^2\phi\right]^{-1}, \quad (68)$$

where dimensionless parameter $\eta = E_A/T_0(Q_0)$ and we introduce

$$\gamma(z) = \beta_1 + \mathcal{J}_1^2(r_0)z^2\frac{\zeta(4,\frac{1}{2}+z)}{\zeta(2,\frac{1}{2}+z)}. \quad (69)$$

Here constant $\beta_1 \approx 2.58$. After integration over absolute value of vector $\mathbf{Q}$ we find that in Eqs.(58), (59), (61) and (64) the following substitution should be used

$$f(\phi)\Delta^2 \rightarrow \frac{r_0}{4\pi N}\zeta\left(2,\frac{1}{2}+\frac{1}{4\pi T_c\tau}\right)\left[\gamma\left(\frac{1}{4\pi T_c\tau}\right)\right]^{-1/2}$$
$$\times \sqrt{\frac{T_c}{T-T_c}}\int_0^{2\pi}\frac{d\phi}{2\pi}\frac{f(\phi)}{\sqrt{1+\frac{\eta T_c}{T-T_c}\sin^2\phi}}. \quad (70)$$

It is worthwhile to mention that in order to obtain the result for $T < T_c$ from the known result for $T > T_c$ we should substitute $2(T_c-T_c)/T_c$ for $(T-T_c)/T_c$ as usual.

B. Fluctuational correction to the anisotropic conductivity $\sigma_{ab}^{(anis)}$

Integrating over angle ϕ in Eq.(70) for $f(\phi) = \cos 2\phi$, we obtain the following fluctuational corrections to the anisotropic part of the conductivity tensor above and below T_c

$$\left.\begin{array}{c}\delta\sigma_{xx}^{(anis-f)}\\\delta\sigma_{yy}^{(anis-f)}\end{array}\right\} = \mp r_0\mathcal{J}_1^2(r_0)H\left(\frac{1}{4\pi T_c\tau}\right)\sqrt{\frac{T_c}{T-T_c}}$$
$$\times \begin{cases}\frac{1}{\sqrt{2}}F_A\left(\frac{\eta T_c}{2(T_c-T)}\right), & T < T_c,\\ F_A\left(\frac{\eta T_c}{T-T_c}\right), & T > T_c.\end{cases} \quad (71)$$

Here function $H(z)$ is determined by the function $h(z)$ as

$$H(z) = \frac{\zeta(2,\frac{1}{2}+z)h(z)}{\sqrt{\gamma(z)}} = \begin{cases}\frac{2\pi^4}{3\sqrt{\beta_1}}z^3, & z \ll 1,\\ \frac{\sqrt{3}}{z\sqrt{\mathcal{J}_1^2(r_0)+3\beta_1}}, & z \gg 1.\end{cases} \quad (72)$$

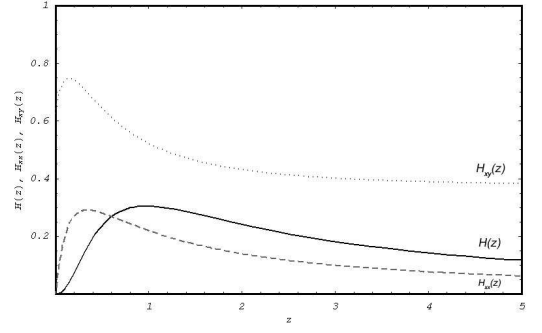FIG. 4: Functions $H(z)$, $H_{xx}(z)$ and $H_{xy}(z)$.

We note that the function $h(z)$ has monotonic growth whereas function $\zeta\left(2,\frac{1}{2}+z\right)/\sqrt{\gamma(z)}$ monotonically decreases. As a result the function $H(z)$ has maximum at $z \approx 0.97$ (see Fig. 4). Function $F_A(x)$ involves complete elliptic functions of the first and second kind

$$F_A(x) = \frac{2}{\pi}\left[\left(1+\frac{2}{x}\right)K(i\sqrt{x}) - \frac{2}{x}E(i\sqrt{x})\right]$$
$$= \begin{cases}\frac{x}{8}, & x \ll 1,\\ \frac{\pi}{\sqrt{x}}\ln 16e^{-4}x, & x \gg 1.\end{cases} \quad (73)$$

Integration over angle ϕ in Eq.(70) with $f(\phi) = \sin 2\phi$ vanishes. Thus, fluctuational corrections to the anisotropic part of σ_{xy} and σ_{yx} conductivities are absent if the UCDW is oriented at the angle $\phi = 0$ with respect to the x axis. We mention that in this case the mean-field contribution to σ_{xy} and σ_{yx} vanishes as well. Consequently, the off-diagonal components of the conductivity tensor are isotropic for $\phi = 0$.

It is worthwhile to emphasize that Eq.(71) constitutes one the main results of the present paper.

C. Fluctuational correction to the anisotropic conductivity $\sigma_{ab}^{(isot)}$

Integrating over angle ϕ in Eq.(70) with $f(\phi) = 1$, we obtain the following fluctuational corrections to the isotropic part of the conductivity tensor above and below T_c

$$\delta\sigma_{xx}^{(isot-f)} = -r_0 H_{xx}\left(\frac{1}{4\pi T_c\tau}\right)\sqrt{\frac{T_c}{T-T_c}} \quad (74)$$
$$\times \begin{cases}\frac{1}{\sqrt{2}}F_I\left(\frac{\eta T_c}{2(T_c-T)}\right), & T < T_c,\\ F_I\left(\frac{\eta T_c}{T-T_c}\right), & T > T_c,\end{cases}$$

and

$$\delta\sigma_{xy}^{(\text{isot}-\text{f})} = -\frac{2\pi r_0 T_c}{\omega_H} H_{xy}\left(\frac{1}{4\pi T_c \tau}\right)\sqrt{\frac{T_c}{T - T_c}} \quad (75)$$

$$\times \begin{cases} \dfrac{1}{\sqrt{2}} F_I\left(\dfrac{\eta T_c}{2(T_c - T)}\right), & T < T_c, \\ F_I\left(\dfrac{\eta T_c}{T - T_c}\right), & T > T_c. \end{cases}$$

Here functions $H_{xx}(z)$ and $H_{xy}(z)$ can be expressed via functions $h_{xx}(z)$ and $h_{xy}(z)$ as follows

$$H_{xx}(z) = \frac{\zeta(2, \frac{1}{2} + z) h_{xx}(z)}{\sqrt{\gamma(z)}} \quad (76)$$

$$= \begin{cases} \dfrac{\pi^2}{3\sqrt{\beta_1}} z, & z \ll 1, \\ \dfrac{\sqrt{3}(1 + 4\mathcal{J}_1^2(r_0))}{4\sqrt{\mathcal{J}_1^2(r_0) + 3\beta_1}}\dfrac{1}{z}, & z \gg 1, \end{cases}$$

$$H_{xy}(z) = \frac{\zeta(2, \frac{1}{2} + z) h_{xy}(z)}{\sqrt{\gamma(z)}} \quad (77)$$

$$= \begin{cases} \dfrac{1}{\sqrt{\beta_1}}\left(1 - \dfrac{\pi^2 - 2\psi''(\frac{1}{2})}{\pi^2} z\right), & z \ll 1, \\ \dfrac{3\pi - 8\mathcal{J}_1^2(r_0)}{4\sqrt{3}\sqrt{\mathcal{J}_1^2(r_0) + 3\beta_1}}, & z \gg 1. \end{cases}$$

We mention that functions $H_{xx}(z)$ and $H_{xy}(z)$ have maximum at z equal to 0.34 and 0.16, respectively, as it is shown in Fig. 4. Function $F_I(x)$ involves complete elliptic integral of the first kind

$$F_I(x) = \frac{2}{\pi} K(i\sqrt{x}) = \begin{cases} 1, & x \ll 1, \\ \dfrac{1}{\pi\sqrt{x}} \ln 16x, & x \gg 1. \end{cases} \quad (78)$$

We emphasize that Eqs.(76)-(77) are one of the main results of the present paper.

D.   Limit of applicability of Eqs.(71), (76) and (77)

Eqs.(71), (75) and (76) have singularity at $T \to T_c$. It indicates that the results are not applicable near T_c. The limit of their applicability is determined by the condition that fluctuational corrections (71), (75) and (76) are still small as compared to conductivity of the liquid state equal to[1]

$$\sigma_{xx}^{(0)} = \frac{2N}{\pi}, \qquad \sigma_{xy}^{(0)} = N. \quad (79)$$

Below we prove that the condition of smallness of fluctuational corrections and the condition $|T_c - T| \ll T_c$ are compatible. Let us first consider Eq.(75). By using
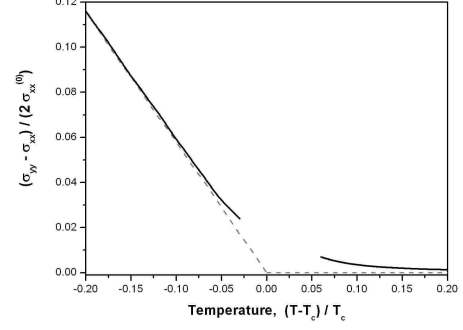

FIG. 5: The dependence of anisotropic part of conductivity $(\sigma_{yy} - \sigma_{xx})/2\sigma_{xx}^{(0)}$ on temperature for $1/4\pi T_c \tau = 0.24$, $\eta = 0.01$ and $N = 2$. Dashed line corresponds to mean-field result (58).

the facts that $\max F_I(x) = 1$ and $\max H_{xx}(z) \approx 0.3$ (see Fig. 4) and Eq.(79), we obtain

$$1 \gg \frac{|T_c - T|}{T_c} \gg N^{-2}. \quad (80)$$

As one can see, Eq.(80) is fulfilled in the limit $N \gg 1$. Analysis of Eq.(71) results in similar non-equality with 0.1 instead of 1 in the right hand side. Eq.(76) contains additional small factor T_c/ω_H. Therefore, the condition of applicability for results (71), (75) and (76) is given by Eq.(80).

V.   DISCUSSION

In the previous sections we derived a number of results for the conductivity tensor of two-dimensional interacting electrons on a half-filled high Landau level. We demonstrated that below temperature T_c of transition from the liquid to the UCDW state the anisotropic part of the conductivity tensor emerges. At $(T_c - T)/T_c \ll 1$ the anisotropic part is proportional to deviation of temperature from T_c. As it is shown in Fig. 5, it results in a cusp of temperature dependence of the conductivity at $T = T_c$. Fluctuations of the order parameter above and below transition temperature T_c smooth out the cusp (as it is shown in Fig. 5).

Results discussed above have been derived in the case of the white-noise random potential. In the case of a random potential with correlation length d arbitrary related with the magnetic length l_H, we can state that temperature dependence of the results above remain the same whereas the functions h_{ab} and H_{ab} become to depend not only on $1/4\pi T_c \tau$ but on ratio d/l_H also.

In experiments[2,3] magnetoresistance R_{xx} and R_{yy} as functions of temperature at filling factor $\nu = 9/2$ have been investigated. Unfortunately, the detailed temperature dependence near the point at which R_{xx} and R_{yy}

become to deviate from each other did not investigated. Nevertheless, results reported in Refs.[2,3] confirm the linear dependence of magnetoresistance R_{xx} and R_{yy} on temperature in the certain range of temperatures not too close to the T_c. Therefore, the detailed investigation of temperature dependence of magnetoresistance are needed in the future.

The results (58) and (59) for the angle dependence of the anisotropic part of the conductivity tensor qualitatively describe the results of experiments[2,3,9]. If a current runs in the direction of charge density modulation, the conductivity (resistance) is less (more) than the conductivity (resistance) for the case of current running in the direction perpendicular to charge density modulation. The Hall conductivity (resistance) remains roughly the same.

The UCDW state exists also near half-filled high Landau level. In the case of $\mu_N \neq 0$, the functions h_{ab} and H_{ab} are dependent not only on parameter $1/4\pi T_c\tau$ but on ratio μ_N/T_c also. Increasing the chemical potential μ_N we transform the UCDW state to the triangular CDW state characterized by vectors $\mathbf{Q}_1$, $\mathbf{Q}_2$ and $\mathbf{Q}_3$ directed at angles ϕ, $\phi + 2\pi/3$ and $\phi + 4\pi/3$ with respect to the x axis, respectively. However, due to the identity

$$1 + e^{2\pi i/3} + e^{-2\pi i/3} = 0 \tag{81}$$

the contribution from three vectors $\mathbf{Q}_i$ to the anisotropic part of conductivity tensor vanishes. Thus, the conductivity tensor of the triangular CDW state remains isotropic in the approximation which is of the second order in the CDW order parameter. From general physical arguments it is clear that the conductivity tensor of the triangular CDW state should be isotropic.

It is worthwhile to mention that from physical point of view the anisotropic part $\sigma_{ab}^{(\text{anis})}$ of conductivity appears due to the existence of the induced anisotropic potential $\lambda(\mathbf{r})$ in action (46). Anisotropic resistance of two-dimensional electrons in a weak magnetic field in the presence of unidirectional periodic potential has been measured at several Kelvins in heterostructures with mobility $\mu_0 \sim 10^6\,cm^2/Vs$ by Weiss, von Klitzing, Ploog, and Weimann fifteen years ago[31]. Theoretically, the effect of unidirectional periodic potential on the conductivity tensor of two-dimensional electrons in a weak magnetic field has been investigated with the help of both the kinetic equation[32,33] and the diagrammatic technique[34,35,36]. However, the case considered in the present paper did not analyze in the Refs.[32,33,34,35,36]. The important difference of the induced potential $\lambda(\mathbf{r})$ from external periodic potential is that the induced potential $\lambda(\mathbf{r})$ appears on the Nth Landau level only whereas external potential scatters electrons on all Landau levels.

VI. CONCLUSION

We obtained the conductivity tensor of two-dimensional electrons in the presence of weak disorder

and weak magnetic field at half-filled high Landau level where the UCDW state exists. In the framework of the order parameter expansion we derived that at $T_c - T \ll T_c$ anisotropic part of the conductivity tensor proportional to $(T_c - T)/T_c$ emerges. Also we demonstrated that the order parameter fluctuations result in additional anisotropic contributions near T_c to the conductivity tensor that wash out the mean-field cusp at $T = T_c$. The results obtained are in agreement with the experimental findings.

APPENDIX A: CALCULATION OF CHARACTERISTIC ENERGY T_1

According to Eqs.(18) and (37) the $T_1(q)$ is given by

$$
\begin{aligned}
T_1(q) = & -n_L e^{i\phi}\Bigg(U_{\text{scr}}(q)\mathcal{J}_0(qR_c)\mathcal{J}_1(qR_c)e^{-i\phi} \\
& - \sqrt{2N}\int \frac{d\boldsymbol{p}}{(2\pi)^2 n_L}e^{-i\mathbf{q}\boldsymbol{p}l_H^2}U_{\text{scr}}(p)\frac{p_x - ip_y}{p^2 l_H} \\
& \times L_{NN}\left(\frac{p^2 l_H^2}{2}\right)L_{N,N-1}\left(\frac{p^2 l_H^2}{2}\right)e^{-p^2 l_H^2/2}\Bigg),
\end{aligned}
\tag{A1}
$$

where we use the following result[1]

$$
\begin{aligned}
F_{N,N-1}(\mathbf{q}) &= \sqrt{2N}\frac{q_x - iq_y}{q^2 l_H}e^{-q^2 l_H^2/4}L_{N,N-1}\left(\frac{q^2 l_H^2}{2}\right) \\
&\approx e^{-i\phi}\mathcal{J}_1(qR_c),\ qR_c \ll 2N.
\end{aligned}
\tag{A2}
$$

The characteristic energy $T_1 = T_1(Q_0)$ is given by

$$
T_1 = \frac{r_s\omega_H}{4\sqrt{2}}\int_0^{4N}\frac{dx}{\tilde{\epsilon}(x)}\frac{\mathcal{J}_1(4Nx)\mathcal{J}_0(4Nx)}{\sqrt{1-x^2}}\mathcal{J}_1(2r_0 x),
\tag{A3}
$$

where

$$
\tilde{\epsilon}(x) = 1 + \frac{r_s}{x\sqrt{2}}(1 - \mathcal{J}_0^2(4Nx)).
\tag{A4}
$$

Performing calculation of the integral, we find

$$
T_1 = \frac{r_s\omega_H}{16\pi N\sqrt{2}}\left[\frac{r_0}{2}\ln\left(1 + \frac{1}{\sqrt{2}r_0 r_s}\right) + \frac{c_1}{1 + \sqrt{2}r_0 r_s}\right],
\tag{A5}
$$

where constant c_1 equals

$$c_1 = \sqrt{\frac{r_0}{\pi}} \int\limits_{1/2r_0}^{1} \frac{dx}{x\sqrt{x(1-x^2)}} \sin\left(2r_0 x - \frac{\pi}{4}\right) \approx 1.097.$$

$$(A6)$$

As we can see from Eq.(A5) the characteristic energy $T_1 \sim T_0/N$ as we mentioned above.

APPENDIX B: CALCULATION OF THE $I_{p_1 p_2 p_3 p_4}(\mathbf{Q}_0)$

Using definition (55) and Eq.(A2), we obtain in the limit $N \gg 1$

$$I_{N,N-1,N-1,N} = I_{N,N+1,N+1,N} = gn_L N \int\limits_0^\infty \frac{dx}{x} e^{-x}$$
$$\times \left[L_N^{-1}(x)\right]^2 \mathcal{J}_0(r_0\sqrt{2x}), \qquad (B1)$$

$$I_{N,N-1,N,N+1} = I_{N,N+1,N,N-1} = gn_L N \int\limits_0^\infty \frac{dx}{x} e^{-x}$$
$$\times L_N^{-1}(x)L_{N+1}^{-1}(x)\mathcal{J}_0(r_0\sqrt{2x}),$$

$$I_{N,N,N,N-1} = -I_{N,N,N,N+1} = gn_L e^{i\phi}\sqrt{N} \int\limits_0^\infty \frac{dx}{\sqrt{x}} e^{-x}$$
$$\times L_N^{-1}(x)L_N(x)\mathcal{J}_1(r_0\sqrt{2x}),$$

$$I_{N,N+1,N,N} = -I_{N,N+1,N,N} = gn_L e^{-i\phi}\sqrt{N} \int\limits_0^\infty \frac{dx}{\sqrt{x}}$$
$$\times e^{-x} L_{N+1}^{-1}(x)L_N(x)\mathcal{J}_1(r_0\sqrt{2x}),$$

$$I_{N,N-1,N,N-1} = I_{N,N+1,N,N+1}^* = gn_L e^{2i\phi} N \int\limits_0^\infty \frac{dx}{x} e^{-x}$$
$$\times \left[L_N^{-1}(x)\right]^2 \mathcal{J}_2(r_0\sqrt{2x}),$$

$$I_{N,N-1,N+1,N} = I_{N,N+1,N-1,N}^* = gn_L e^{2i\phi} N \int\limits_0^\infty \frac{dx}{x} e^{-x}$$
$$\times L_N^{-1}(x)L_{N+1}^{-1}(x)\mathcal{J}_2(r_0\sqrt{2x}).$$

With a help of asymptotic expression for Laguerre polynomial[37]

$$L_N^\alpha(x) \simeq \frac{1}{\sqrt{\pi x}} e^{x/2} \left(\frac{n}{x}\right)^{(2\alpha-1)/4} \cos\left(2\sqrt{nx} - \frac{\alpha\pi}{2} - \frac{\pi}{4}\right),$$
$$N \gg 1, \qquad (B2)$$

we find

$$I_{N,N-1,N-1,N} = I_{N,N+1,N+1,N} \qquad (B3)$$
$$= gn_L \frac{2}{\pi} \int\limits_0^1 dx \frac{\mathcal{J}_0(2r_0 x)}{\sqrt{1-x^2}},$$

$$I_{N,N-1,N,N+1} = I_{N,N+1,N,N-1}$$
$$= gn_L \frac{2}{\pi} \int\limits_0^1 dx (1-2x^2) \frac{\mathcal{J}_0(2r_0 x)}{\sqrt{1-x^2}},$$

$$I_{N,N,N,N-1} = -I_{N,N,N,N+1}$$
$$= gn_L e^{i\phi} \frac{2}{\pi} \int\limits_0^1 dx\, x \frac{\mathcal{J}_1(2r_0 x)}{\sqrt{1-x^2}},$$

$$I_{N,N+1,N,N} = -I_{N,N+1,N,N}$$
$$= gn_L e^{-i\phi} \frac{2}{\pi} \int\limits_0^1 dx\, x \frac{\mathcal{J}_1(2r_0 x)}{\sqrt{1-x^2}},$$

$$I_{N,N-1,N,N-1} = I_{N,N+1,N,N+1}^*$$
$$= gn_L e^{2i\phi} \frac{2}{\pi} \int\limits_0^1 dx \frac{\mathcal{J}_2(2r_0 x)}{\sqrt{1-x^2}},$$

$$I_{N,N-1,N+1,N} = I_{N,N+1,N-1,N}^*$$
$$= gn_L e^{2i\phi} \frac{2}{\pi} \int\limits_0^1 dx (1-2x^2) \frac{\mathcal{J}_2(2r_0 x)}{\sqrt{1-x^2}}.$$

The integrals can be evaluated by using the following equality[37]

$$\frac{2}{\pi} \int\limits_0^{\pi/2} d\phi \cos(2\mu\phi) \mathcal{J}_{2\nu}(2r_0 \cos\phi) = \mathcal{J}_{\nu+\mu}(r_0)\mathcal{J}_{\nu-\mu}(r_0).$$

$$(B4)$$

Finally, it yields the results presented in Table I.

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